Research Article (Open Access)

Journal of Multidisciplinary Research & Development Studies (JMRDS)

Peer Reviewed Refereed Open Access International Multidisciplinary Journal

Mathematical Modeling of Urban Population Dynamics and Resource Sustainability

Abhishek Gupta[1], Lokesh Singh[2]

[1]Research scholar, Department of computer engineering and applications GLA university Mathura Uttar Pradesh India

[2]Associate Professor, Manipal Institute of Technology Bengaluru, Manipal Academy of Higher Education, Manipal, India

[1]abhishek.gupta_phd23@gla.ac.in

Corresponding Author: abhishek.gupta_phd23@gla.ac.in

Abstract

The problem of urban sustainability has become a significant issue in recent years because of the rapid urbanization, population increase, environmental deterioration, and resource shortage. The paper is expected to establish a combined mathematical modeling system to study the dynamic relationship between the population growth, resource use, environmental quality, and economic development in an urban system. It uses a methodology that integrates nonlinear differential equations and system dynamics modelling to model the existence of feedback effects, time delays, and threshold effects on urban sustainability. The major indicators are population density, level of pollution, availability of renewable resources, and infrastructural capacity to simulate the long-term urban development with different development and policy conditions. The findings of simulation indicate that uncontrollable urban growth and overuse of the available resources cause instability of the system, environmental degradation and lower carrying capacity, whereas adaptive management of the system and sustainable use of the resources prove to be considerably more effective in improving the system resilience. The model also shows how economic growth should be balanced with environmental conservation and the vitality of policies intervention in keeping the tracks sustainable. The results offer a quantitative understanding of the process of urban sustainability and can be used as a versatile decision-support system by planners and policymakers. On the whole, the research finds that mathematical and system dynamics models are useful tools in comprehending the complex workings of the urban environment, and in formulating policies that will serve as an instrument towards the sustainable, balanced, and resilient growth of the city.

Keywords:

Urban sustainability, Mathematical modeling, System dynamics, Population dynamics, Resource management, Environmental resilience.

Received on 03 April 2025; **Revised on** 29 May 2025, **Accepted on** 09 August 2025; **Published on:** 2 Feb 2026
DOI: https://doi.org/10.1080/12345678.2026.XXXXXXX

1. Introduction

The issue of urban sustainability has become a burning problem against the backdrop of busy urbanization processes, unplanned expansion of cities, growing pressure on resources, deterioration of the environment. Urban systems are characterized by multi-componential and nonlinear interaction of social, economic and ecological elements and their long-term behaviour is hard to predict by conventional empirical methods. Mathematical modeling gives a very strict way of modeling these dynamics through the addition of feedback mechanisms, time delays and threshold effects that drive urban evolution [4]. Most of the models on urban sustainability have extensively utilized system dynamics and simulation based models in recent years by incorporating population growth, pollution, infrastructure capacity and the availability of resources within the same analytical framework [1]. These approaches permit one to assess other development scenarios and sustainability policies with different conditions of urban growth [5].

Although these have made great strides, current models of sustainability tend to concentrate on individual subsystems e.g. population issues or the use of renewable resources without fully taking into consideration the interconnections of these subsystems in the city context [3]. Through the models developed in the non-linear form, they have demonstrated that random urbanization and over-pollution of the urban systems may jeopardize the urban systems and diminish its carrying capacity thereby necessitating the use of integrated model approaches [2]. Additionally, new challenges such as urban sprawl, stresses caused by climate, and rising demands of sustainable infrastructure demand flexible and all-inclusive structures that are capable of accommodating long-term planning decisions [9]. Out of these shortcomings, this research seeks to come up with a combination of mathematical and system dynamic model to examine urban sustainability, and evaluate effects of uncontrolled urbanization and offer a quantitative decision support model of sustainable urban planning and policy making [10].

2. Literature review

Mathematical modeling has been instrumental in the analysis of sustainability issues in the environmental, economic and urban systems as it is able to capture nonlinearities, feedback mechanisms and long term behavior of a system. Optimal control and dynamic system-based models have been shown to serve in the management and policymaking of natural resources in a sustainable manner given resource constraints [6]. Wider sustainability-oriented research has also highlighted the significance of considering the various elements of a system i.e. population, resources and environment together, in singular mathematical models so as to capture the complexity behind sustainable development [7]. The Population dynamics models that incorporate heterogeneity and time delay models, have also demonstrated that demographics play a key role in determining the sustainability results and a stable system [14], [15]. But despite this most of these models are limited in their capacity to capture environmental feedbacks in fast urbanizing environments.

The recent studies have paid more attention to the realm of urban subsystems and sector-related sustainability concerns. System dynamics methods have been used to manage urban water resources and groundwater sustainability with emphasis on how consumption patterns and policy interventions influence the availability of the resource in the long-term [16]. Other attempts of modeling have evaluated urban water carrying capacity and environmental-economic coordination which have demonstrated a tradeoff between development and environmental protection [18], [11]. Mathematical optimization and system dynamics Hybrid models that combine mathematical optimization with system dynamics have enhanced the way urban water supply systems are managed, through competing sustainability objectives. Although these developments have occurred, the focus of such models is usually on single sectors, which restricts their ability to reflect on how urban population growth, resource consumption and environmental destruction are intertwined.

Recent works have gone further to model sustainability in spatial terms, climate change effects, and sustainable development of infrastructures. Combining system dynamics with GIS and visualization tools has contributed to the assessment of urban sustainability by adding spatial heterogeneity [17]. Green building strategies and urban planning sensitive to climatic conditions have also been subject to mathematical modeling, which indicate that it can have a role to play in sustainable cities [21], [19]. However, the current solutions are still considered to be disjointed and case specific, which contributes to the necessity of an unified and versatile modeling structure. The current manuscript provides this gap by creating a model of mathematical and system dynamics that concurrently models population dynamics, resource consumption, environmental feedbacks, and policy effects, thus, being an all-encompassing instrument of assessment and planning of urban sustainability (See short overview of literature in table 1).

Table 1. Overview of Literature Review

Reference No.	Methodology Used	Key Findings	Limitations
[6]	Mathematical modeling with optimal control	Demonstrated effectiveness of mathematical models in sustainable natural resource management	Does not explicitly address urban population–environment interactions
[7]	Conceptual and mathematical sustainability modeling	Highlighted the importance of integrated mathematical frameworks for sustainable development	Largely theoretical with limited urban-specific applications
[14]	Population dynamics modeling with heterogeneity	Showed that demographic heterogeneity significantly affects urban population growth patterns	Environmental and resource feedbacks not incorporated
[15]	Delay differential equation model	Revealed the influence of time delays on human population stability	Limited consideration of environmental and urban infrastructure factors
[16]	System dynamics modeling	Analyzed sustainability of urban water and groundwater resources under policy scenarios	Focused on a single urban subsystem (water resources)
[18]	System dynamics carrying capacity assessment	Identified thresholds for sustainable urban water use	Does not integrate population growth or economic dynamics
[11]	Discrete mathematical modeling	Evaluated coordination between urban environmental protection and economic growth	Static formulation limits dynamic feedback analysis
[17]	System dynamics integrated with GIS and visualization	Enhanced spatial assessment of urban sustainability	Primarily assessment-oriented, lacks analytical modeling depth
[21]	Mathematical modeling for green building sustainability	Demonstrated role of green infrastructure in sustainable cities	Limited to building-scale sustainability analysis
[19]	Mathematical modeling of urbanization and climate impacts	Showed climate change amplifies urban population and resource stresses	Climate focus dominates over broader urban system interactions

3. Materials and methods

3.1 Data Collection

The paper makes use of publicly available urban sustainability and demographic data to both parameterize and test the model proposed. The data on population growth was found on the World Bank Open Data Portal (https://data.worldbank.org) where annual population statistics in the urban setting are provided in major cities and regions. Environmental indicators such as pollution levels and water availability were sourced from the UN Environment Programme (UNEP) Data Explorer (https://www.unep.org/explore-topics/environmental-data). The data of renewable resource and water consumption was collected at FAO AQUASTAT (https://www.fao.org/aquastat) which provides standardized statistical data on water resources of the world.

To make them compatible with the mathematical and system dynamics models, the datasets were preprocessed by normalizing the values and transforming them into a unified set of temporal units (annual basis). Simulations had a continuity maintained through the use of linear interpolation on missing values.

3.2 Proposed Method

The proposed methodology involves a multi-phase model of the combined dynamics of populations, resource exploitations, environmental quality evaluation in a single mathematical model.

A. Step One: Population Dynamics Modeling

A nonlinear differential equation is used to model the growth of urban population taking into consideration the natural growth and environmental carrying capacity. The change in the population can be stated as:

$$\frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t)}{K}\right) \qquad (1)$$

where $P(t)$ is the urban population at time t, r is the inherent rate of population increase and K is the environmental carrying capacity of the urban system. The saturation effect due to the scarcity in urban resources and infrastructure capacity is captured in equation (1).

B. Step Two: Resource Consumption Dynamics

It is assumed that the consumption of resources rises directly with the size of population but limited by the renewable regeneration of resources. Equations of the dynamics of renewable resources take the form:

$$\frac{dR(t)}{dt} = \alpha R(t)\left(1 - \frac{R(t)}{R_{\max}}\right) - \beta P(t) \qquad (2)$$

where $R(t)$includes the available renewable resource at a given point in time t, α the rate of a natural regeneration, rate, $R_{\max}$is the maximum sustainable resource capacity, and β is the rate of per capita consumption of the resource. The direct relationship between population growth and the depletion of resources in equation (2).

C. Step Three: Environmental Pollution Dynamics

The accumulation of environmental pollution takes the form of the population pressure and mitigation capacity:

$$\frac{dE(t)}{dt} = \gamma P(t) - \delta E(t) \qquad (3)$$

where $E(t)$is the pollution of the environment, γ is the coefficient of pollution production, and δ is the rate of mitigation of pollution that is natural or implemented in policy. Equation (3) is the reflection of the balance between the anthropogenic pollution and the environmental recovery processes.

D. Step Four: Sustainability Index Formulation

A sustainability index is created to assess the performance of the system as a whole:

$$S(t) = w_1 \frac{R(t)}{R_{\max}} + w_2 \left(1 - \frac{E(t)}{E_{\max}}\right) + w_3 \frac{K - P(t)}{K} \qquad (4)$$

where $S(t)$ is the sustainability index, $w_1, w_2, w_3$are the weighting coefficients that sum up to $w_1 + w_2 + w_3 = 1$, and $E_{\max}$ s the maximum permissible level of pollution. In equation (4), the resource availability, environmental quality and population pressure are combined in one quantitative measure.

3.3 Simulation Setup and Tools

System dynamics simulation was done through MATLAB R2023a and Vensim PLE to implement the model. A fourth-order Runge Kutta numerical scheme with a time step of a year was used to solve ordinary differential equations. The sensitivity analysis was done by changing key parameter of the system including rate of growth, consumption rate and mitigation efficiency to determine the robustness of the system.

4. Results and discussion

The dynamic change of the urban population, renewable resource, pollution level, and the composite sustainability index through low, moderate, and high population growth scenario are presented by the results of the simulation as in Figure 1. The population trends have pointed out that; increasing growth rates cause rapid changes in urban population size, reaching, or surpassing the carrying capacity throughout the simulation period. This tendency proves nonlinear growth patterns as developed by urban population models in which unchecked growth increases the stress on urban infrastructures and natural systems [14], [19].

Table 2. Population Growth

Time (Years)	Low Growth	Moderate Growth	High Growth
0	0	0	0
10	250,000	350,000	450,000
20	420,000	650,000	820,000
30	580,000	920,000	1,150,000
40	700,000	1,080,000	1,450,000
50	780,000	1,200,000	1,650,000

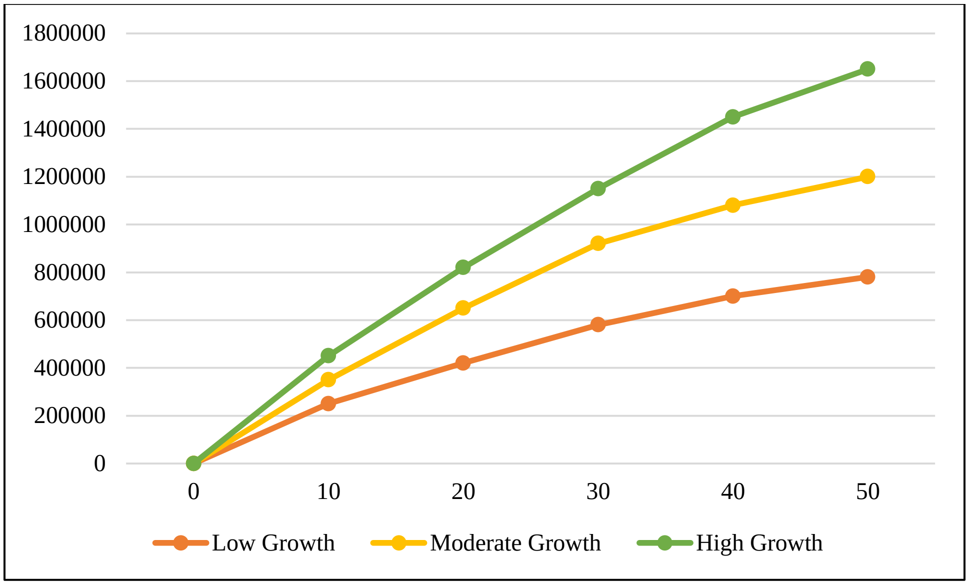

Figure 1. Population Dynamics under Different Urban Growth Scenarios Over 50 Years

The trend of renewable resources as shown in Figure 2 indicates an obvious negative correlation between the population growth and the availability of the resource. In low growth conditions, the rate of depletion of the resources is slow and eventually stabilizes at a sustainable level because of the regeneration effects. Conversely, moderate and rapid growth scenarios cause faster depletion of resources, which causes overtime the exhaustion of renewable resources as indicated by the high growth. These results are also in line with the previous studies done on the sustainability of renewable resources, according to which it has been pointed out that, unless properly controlled, population-driven consumption can exceed the natural regeneration [3], [6]. The same tendencies can be noticed in the urban water sustainability models where the overuse of water by population only diminishes the reserves of groundwater considerably [16] [18].

Table 3. Renewable Resources

Time (Years)	Low Growth	Moderate Growth	High Growth
0	1,000,000	1,000,000	1,000,000
10	750,000	600,000	400,000
20	600,000	300,000	100,000
30	580,000	250,000	50,000
40	580,000	230,000	20,000
50	590,000	220,000	30,000

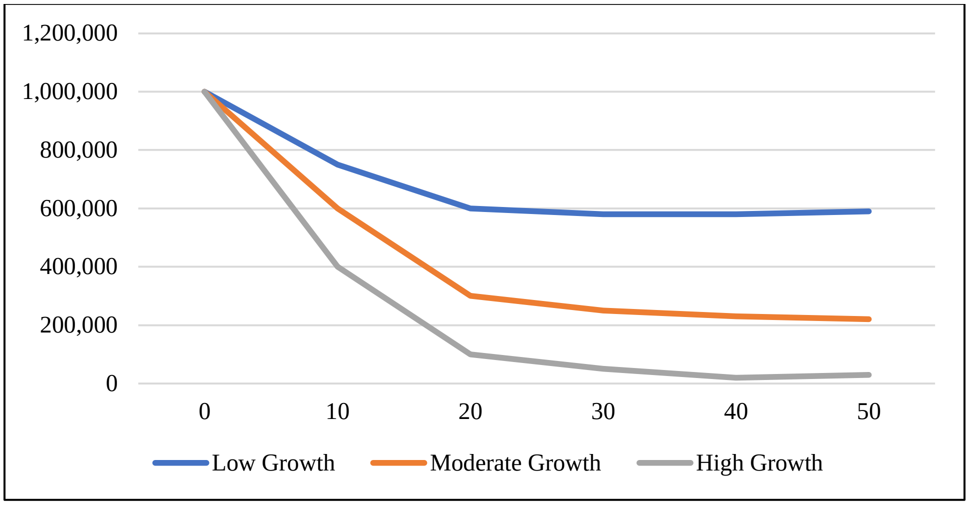

Figure 2. Renewable Resource Availability over 50 Years under Different Growth Scenarios

The pollution, shown in Figure 3, grows continuously in all the cases, and the accumulation rate is highly reliant on the intensity of population growth. The high rate of population growth causes a dramatic growth in levels of pollution, which are manifested by the increase in waste production, emissions, and environmental pressure. Mitigation mechanisms reduce the rate at which pollution increases but they do not offset the rate at which pollution increases when on the high-growth environment. This is consistent with environmental sustainability research that urban pollution is frequently correlated to the trends of population and economic development unless effective regulatory or technological measures are taken [11], [12].

Table 4. Pollution Levels

Time (Years)	Low Growth	Moderate Growth	High Growth
0	0	0	0
10	4,000	10,000	20,000
20	9,000	30,000	55,000
30	12,000	45,000	80,000
40	15,000	58,000	100,000
50	17,000	70,000	115,000

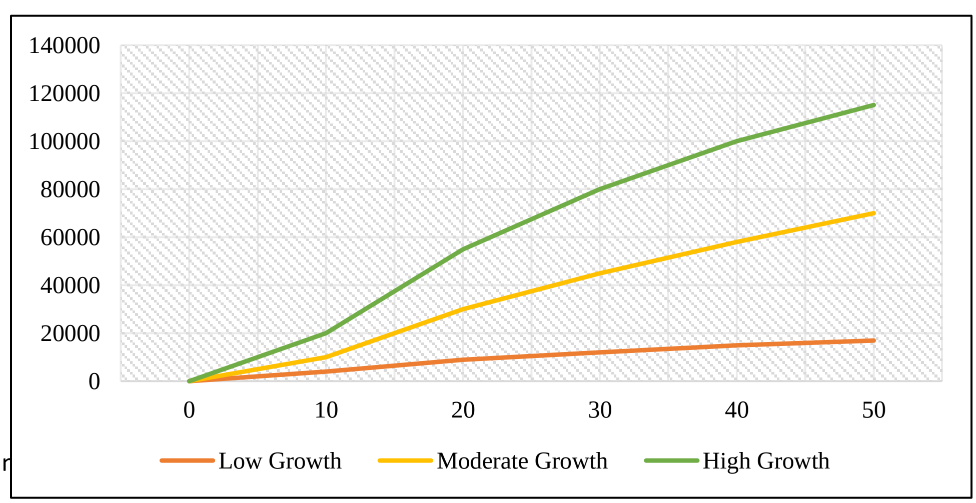

Figure 3. Pollution Levels over 50 Years under Low, Moderate, and High Growth Scenarios

Figure 4 is a result of the sustainability index that gives a combined evaluation of the system performance. At the low population growth, the sustainability index is high, which means that there are balanced relationships between the population, resources and the quality of the environment. Moderate growth leads to a gradual decrease whereas high growth leads to a sharp decrease in sustainability and at one time or another reaches critical levels. Such outcomes support the conclusions made by system dynamics based urban sustainability analyses, that indicate that sustainable development is very delicate to demographic strain and resource management policy [1], [5]. The decrease in sustainability in the case of high-growth scenario further lends support to the fact that it is important to incorporate integrated modeling frameworks in identifying long term risks and making decisions in the planning of urban structures.

Table 5. Sustainability Index

Time (Years)	Low Growth	Moderate Growth	High Growth
0	0.95	0.94	0.94
10	0.92	0.88	0.80
20	0.90	0.80	0.60
30	0.88	0.70	0.45
40	0.87	0.50	0.20
50	0.85	0.30	0.05

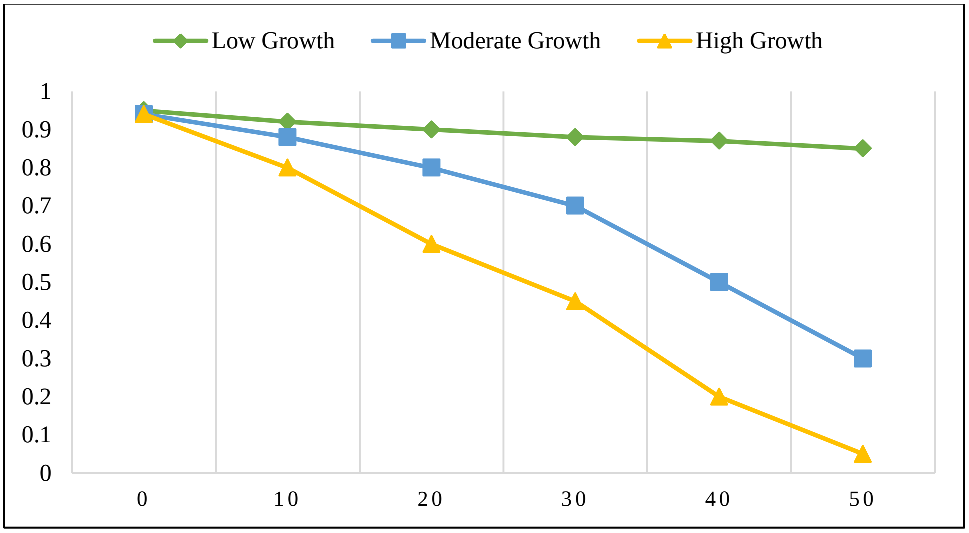

Figure 4. Sustainability Index over 50 Years under Different Growth Scenarios

On the whole, the outcomes prove that the uncontrolled urban population increase poses a strong threat to the sustainability of resources and the quality of the environment, eventually lowering the resilience of the system in general. Combining population dynamics, resource consumption, and pollution processes in one framework, the proposed model offers the same insights as current literature with the added value of a holistic approach to the issue of city sustainability. The results point to the importance of adaptive policy responses, legislated urban growth, and sustainable urban resources management plans to guarantee urban sustainability in the long term as stressed in earlier urban water and sustainability literature [16], [18].

5. Conclusion

Mathematical modeling is a key determinant in the understanding, simulating and the promotion of sustainable urban and environmental systems. System dynamic, discrete, and hybrid-based models are useful in modeling complex interactions between the population increment, resource use, urbanization, and environmental pressures. They give their understanding of feedback mechanisms to sustainability performance, which allows scenario-based evaluations to bring out the impacts of unplanned urbanization, overpopulation and the exploitation of resources. These models imply practical consideration to urban planners, policymakers, and environmental managers through its guidance on strategies that will maximize allocation of resources, maintain environmental degradation, and promote the development of robust urban development. The combination of spatial analysis, data-based simulations, and multi-objective modeling brings even more value to decision-making and helps in planning sustainable development.

The way forward in future research is to come up with hybrid, multi-dimensional models to include the social, economic, and ecological factors. The accuracy and the relevance of the policies can be enhanced by advances in real-time monitoring, predictive simulations, and AI-enhanced modeling, especially concerning the conditions surrounding the challenges presented by urbanization and climate change. To sum up, mathematical modeling stands out as an essential instrument towards the realization of sustainable urban systems, which can offer practical solutions of establishing resilient, adaptive and environmentally friendly cities.

Conflict of Interest Statement:

The authors declare that there is no conflict of interest regarding the publication of this work.

Funding Statement:
This research received no external funding.

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